B.Sc. (Honours) Examination, 2019
Semester - VI
Statistics
Course : BSC-61
(Statistical Quality Control and Demography)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions **two** from each group

Group A

1. a) Describe the following terms:
   Process control, chance causes, assignable causes, $3\sigma -$ limits, control charts
   b) Describe, in detail, the construction of control chart for number of defective items 6+4
2. a) Discuss the following concepts in connection with sampling inspection plan:
   Consumer's risk, producer's risk, AOQL, OC curve and ASN curve
   b) For a single sampling plan, obtain the expressions for the OC and ASN functions. 5+5
3. a) Describe the construction of $(\bar{X}, R)$ chart for varying sample size.
   b) Describe the technique of sampling inspection by variables in the normal distribution case. 5+5

Group B

4. Distinguish between –
   a) Stable population and stationary population
   b) Morbidity incidence rate and morbidity prevalence rate 5+5
5. a) Prove that $\frac{d}{dx}e_x^0 = \mu_x e_x^0 - 1$, symbols having their usual meaning.
   b) What do you mean by the statement: "NRR of a country is 2.7". Explain why NRR is always less than GRR.
6. Describe the method of fitting a Logistic curve by
   a) Rhode's method
   b) Fisher's method 5+5

B.Sc. (Honours) Examination, 2019
Semester-IV
Statistics
Course : CC-9

(Linear Models)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. Let y_1, y_2, y_3 are three uncorrelated random variables having common variance σ^2. If

$$E(y_1) = \theta_1 - \theta_2$$
$$E(y_2) = 2\theta_1 + \theta_2$$
$$E(y_3) = \theta_1 - 2\theta_2$$

Obtain the unbiased estimators of θ_1 and θ_2. Also obtain their variances and the covariances. Are your above obtained estimates have minimum variances? If not then indicated how can you obtain that. 10

2. Consider the Gauss Markov Model

$$E(y_1) = 2\theta_1 + \theta_2$$
$$E(y_2) = \theta_1 - \theta_2$$
$$E(y_3) = \theta_1 - \alpha\theta_2$$

With usual assumptions. Determine α such that the best linear estimartess (BLUEs) of θ_1 and θ_2 are uncorrelated. 10

3. a) Clearly state the assumptions that are made in the analysis of variance. 3
   b) State how violations of these assumptions affect the analysis and how the violations in the assumptions may be avoided. 7
4. Describe how would you test for the relationship between two variables using ANOVA technique. 10
5. Show that for a set of two-way classified data with one observation per cell having the model set up. 10

$$y_{ij} = \mu + \alpha_i + \beta_j + e_{ij}$$

with $\sum_i \alpha_i = \sum_j \beta_j = 0$ and $e_{ij} \overset{iid}{\sim} N(0, \sigma^2)$ the following is true

$$\sum_i \sum_j (y_{ij} - \mu - \alpha_i - \beta_j)^2 = pq(\bar{y}_{00} - \bar{\mu})^2 + q\sum_i (\bar{y}_{i0} - \bar{y}_{00} - \alpha_i)^2$$
$$+ p\sum_j (\bar{y}_{oj} - \bar{y}_{oo} - \beta_j)^2 + \sum_i \sum_j (y_{ij} - \bar{y}_{io} - \bar{y}_{oj} + \bar{y}_{oo})^2$$

Use the above relation to obtain the least square estimates of the parameter of the model. Use this to obtain also SSE, SSA and SSB.

P.T.O.

B.Sc. (Honours) Examination, 2019
Semester-VI
Statistics
Course : BSC-62
(Linear Models and Categorical Data Analysis)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. a) Consider the following model

$$E(y_1) = \beta_1 + \beta_2$$
$$E(y_2) = \beta_1 + \beta_3$$
$$E(y_3) = \beta_3 - \beta_2$$

Where y_1, y_2, y_3 are independent random variables having common variance σ^2. Find the Beast Linear Unbiased Estimator of the function $\beta_1 - \beta_2 + 2\beta_3$. 8

b) State Gausss-Markov theorem with assumptions in the context of estimability of parameters of a linear model. 2

2. Consider the model

$$y_1 = \beta_1 + \epsilon_1$$
$$y_2 = 2\beta_1 - \beta_2 + \epsilon_2$$
$$y_3 = \beta_1 + 2\beta_2 + \epsilon_3$$
where $\epsilon \sim N(0,1)$

Derive an appropriate test for testing the hypothesis $H_0 : \beta_1 = \beta_2$. 10

3. a) Use the technique of analysis of variance for testing whether two regression lines are parallel. 7

b) How would you interpret an observed F which is less than one? 3

4. a) Derive the expected values of mean squares for two way classified data with one observation per cell under fixed effect model. 8

b) Under this model set up is it possible to test the interaction effect? If not, when will that be possible? 2

5. Describe how the ANOVA technique can be applied to test that there is no dependence of y (dependent variable) on the k fixed variables $x_1, x_2, \ldots x_k$ (independent variables). 10

6. a) What is an ANCOVA model? Explain how does it reflect better model adequacy diagnostics. 3

b) Under a one-way lay out with one fixed concomitant variable describe how the equality of factor effects can be tested. 7

7. a) Define pearson's χ^2 Goodness of Fit statistic and like-lihood-Ratio Goodness-of –Fit statistic G^2. Show that G^2 and χ^2 statistics usually have similar values. 5

b) State and prove Fisher's exact test for a 2×2 contingency table. 5

B.Sc. (Honours) Examination, 2019
Semester-IV
Statistics
Course : CC-10
(Statistical Quality Control)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. a) Describe the following terms:
   Process control, chance causes, assignable causes, 3σ – limits, control charts
   b) Describe, in detail, the construction of control chart for number of defective items 6+4
2. a) Discuss the following concepts in connection with sampling inspection plan:
   Consumer's risk, producer's risk, AOQL, OC curve and ASN curve
   b) For a single sampling plan, obtain the expressions for the OC and ASN functions. 5+5
3. a) Describe the construction of $(\bar{X}, R)$ chart for varying sample size.
   b) Describe the technique of sampling inspection by variables in the normal distribution case. 5+5
4. a) Distinguish between –
   i. Process control and product control
   ii. Sample size and rational subgroup
   b) Describe the construction of control chart for number of defects. 4+6
5. Describe the sequential inspection plan in details. Compare it with the single inspection plan and double inspection plan in terms of number of samples required to test. 10
6. What is a double sampling inspection plan? Find the expression of OC and ASN functions for a double inspection plan. 10

Use Separate Answer Scripts for each Unit

Undergraduate Examination 2019

Semester – IV (CBCS)

Mathematics

Generic Elective Course: GEC-4

(Linear Programming Problem and Numerical Methods)

Time: Three Hours **Full Marks: 60**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.

UNIT-I [Linear Programming Problem (Marks: 30)]

Answer ***any three*** questions.

1. a) Find the basic solutions of the set of equations
$$2x_1 + 4x_2 - 2x_3 = 10,$$
$$10x_1 + 3x_2 + 7x_3 = 33.$$ 4

   b) When an L.P.P. has (i) no feasible solution, (ii) unbounded solution? 2

   c) Given $x_1 = 1, x_2 = 3, x_3 = 2$ is a feasible solution of the equations
$$2x_1 + 4x_2 - 2x_3 = 10,$$
$$10x_1 + 3x_2 + 7x_3 = 33.$$
   Reduce the above feasible solution to a basic feasible solution by reduction theorem. 4

2. a) Prove that convex polyhedron is a convex set. 4

   b) Examine if $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 25\}$ is a convex set. Find its extreme points, if any. 3+1

   c) Give an example with justifications of a non-convex set. 2

3. a) Solve graphically the L.P.P.
   Minimize, $z = -2x_1 + x_2$
   Subject to
$$x_1 + x_2 \geq 6,$$
$$3x_1 + 2x_2 \geq 16,$$
$$x_2 \leq 9,\ x_1 \geq 0,\ x_2 \geq 0.$$ 4

   b) Solve by simplex method the L.P.P.
   Maximize, $z = x_1 + 2x_2$
   Subject to
$$x_1 - 5x_2 \leq 10,$$
$$2x_1 - x_2 \geq 2,$$
$$x_1 + x_2 = 10,\ x_1 \geq 0,\ x_2 \geq 0.$$ 6

4. a) Prove that for a primal L.P.P. dual of the dual is the primal itself. 4

   b) Solve the problem
   Minimize, $z = 3x_1 + x_2$
   Subject to
$$2x_1 + x_2 \geq 14,$$
$$x_1 - x_2 \geq 4,\ x_1 \geq 0,\ x_2 \geq 0.$$
   by solving its dual problem with the help of simplex method. 2+4

P.T.O.

5. a) Find the dual of the problem

Maximize, $z = 2x_1 + x_3$

Subject to

$4x_1 - 5x_2 + x_3 = 0,$

$x_1 + 2x_2 + 3x_3 \le 7,\ x_1 \ge 0,\ x_2 \ge 0,$

where x_3 is unrestricted in sign. 3

b) Solve the following balanced T.P. by using VAM to determine the initial B.F.S. 4+3

	D_1	D_2	D_3	D_4	a_i
O_1	4	2	7	-1	27
O_2	3	0	2	4	33
O_3	5	3	4	5	23
O_4	3	5	4	-2	17
b_j	31	24	25	20	100 / 100

UNIT-II [Numerical Methods (Marks: 30)]

Answer ***any three*** questions.

1. a) Obtain the general formula for estimating relative error of a function of n variables. 3
   b) When does the significant error arises leading to a inaccurate value? 2
   c) If $\Delta r = \Delta h = 0.01$, find the absolute and relative errors correct upto two significant figures for $V = \frac{1}{3}\pi r^2 h$, when $r = 2$ and $h = 3$. 3
   d) Find the number of significant figures in
      i) $V_A = 0.3941$ given the absolute error as 0.25×10^{-2} 1
      ii) $V_T = 1.5975$ given the relative error as 0.1×10^{-2} 1

2. a) Establish Newton's forward interpolation formula without error term. 4
   b) Find the missing figure in the following table: 3

x	16	18	20	22	24	26
$f(x)$	39	85	—	151	233	388

   c) If $f(x) = e^{ax}$, then show that $f(0), \Delta f(0), \Delta^2 f(0)$ are in geometric progression. 3

3. a) If $f(x) = \sin x$, then show that $\Delta^2 f(x) = -\left(2\sin\frac{1}{2}\right)^2 E f(x)$, taking $h = 1$. 3
   b) Show that the sum of the Lagrangian function is unity. 3
   c) Using the suitable interpolation formula find the polynomial which passes through the points $(-1,9), (0,5), (2,3)$ and $(5,15)$. 4

a) Derive composite Simpson's one-third rule for the numerical integration $\int_a^b f(x)dx$. 5

b) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\cos x^2}\, dx$, correct upto 4 decimal places taking 10 intervals by
   i) Trapizoidal rule
   ii) Simpson's one-third rule. 5

a) Find the condition of convergence of the fixed point iteration method for the solution of an algebraic or transcendental equation. 3
b) Find the real root of the equation $3x - \cos x - 1 = 0$ by Newton-Rapson method. 2
c) Solve the equation $x^3 - 9x + 1 = 0$ for the root lying between 2 and 3, correct to 3 significant figures. 2
d) Describe briefly Gauss-Jacobi method for solving a linear system of algebraic equations. 3

B.Sc. (Honours) Examination, 2019
Semester-IV
Statistics
Course : CC-8
(Survey Sampling & Indian Official System)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Group - A (Answer **any ten** questions)

1. Answer the following questions by selecting the correct option(s)

(a) The total number of possible sample of size 3 drawn by SRSWR from a population of 6 member is
(i) 20 (ii) 36 (iii) 216 (iv) 729.

(b) How many ways we can collect information from a population?
(i) 1 (ii) 2 (iii) 3 (iv) 4.

(c) An unknown number of minibus in a city suppose to be numbered serially from 1 to N. If the n different minibus you have come across in the city can be assumed to form a random sample, then which of the following is an unbiased estimator of $(N+1)$
(i) $\bar{x}$ (ii) $2\bar{x}-2$ (iii) $2\bar{x}-1$ (iv) $2\bar{x}$.

(d) In simple random sampling without replacement probability that a particular member included in the sample is
(i) $\frac{1}{N}$ (ii) $\frac{1}{n}$ (iii) $\frac{n}{N}$ (iv) $\frac{N}{n}$.

(e) Two basic principles of sample survey are
(i) validity (ii) optimization (iii) complete enumeration (iv) sampling fluctuations.

(f) Which of the following is not a probability sampling
(i) simple random sampling (ii) stratified sampling (iii) systematic sampling (iv) cluster sampling

(g) Identify the sample sizes for equal allocation, proportional allocation and Neyman or optimum allocation
(i) $\frac{k}{n}$ (ii) $\frac{nN_iS_i}{\sum_{i=1}^{k} N_iS_i}$ (iii) $(\frac{n}{N})N_i$ (iv) $\frac{n}{k}$.

(h) Identify the correct inequality
(i) $Var(\bar{y}_{opt}) \leq Var(\bar{y}_{wor}) \leq Var(\bar{y}_{prop})$ (ii) $Var(\bar{y}_{opt}) \leq Var(\bar{y}_{prop}) \leq Var(\bar{y}_{wor})$
(iii) $Var(\bar{y}_{prop}) \leq Var(\bar{y}_{wor}) \leq Var(\bar{y}_{opt})$ (iv) $Var(\bar{y}_{prop}) \leq Var(\bar{y}_{opt}) \leq Var(\bar{y}_{wor})$.

(i) Which of the following estimators are unbiased estimator of population mean
i) $\bar{y}_{wor}$ (ii) $\bar{y}_{wr}$ (iii) $\bar{y}_{st}$ (iv) $\bar{y}_{sys}$.

(j) A statistical measure based on population values, is called
(i) parameter (ii) statistic (iii) estimator (iv) estimate.

(k) Which of the following are the major sources on 'Health Statistics' in India?
1. Sample Registration System (SRS).
2. Civil Registration System (CRS).

P.T.O.

3. National Family Health Survey (NFHS).

Select the correct answer using the code below

i) 1 & 2 (ii) 1 & 3 (iii) 2 & 3 (iv) 1,2 & 3.

(l) The first phase of Population Census is associated with which of the following?

1. House Listing
2. Housing Census.
3. Population Enumeration.

Select the correct answer using the code below

i) 1 & 2 (ii) 2 & 3 (iii) 1 & 3 (iv) 1,2 & 3.

(m) The most common sampling design used by NSSO in its socio-economic surveys is

(i) Simple Random Sampling (ii) Circular Systematic Sampling (iii) Probability Proportional to Size Sampling (iv) Two-stage Stratified Sampling.

(n) Which one of the following organizations of the Government is responsible for collection, compilation and dissemination of Trade Statistics in India?

(i) CSO (ii) Directorate General of Commercial Intelligence and Statistics (iii) Department of Industrial Policy and Promotion (iv) Tariff Commission.

(o) Which of the following are part of mandate of Survey Design and Research Division (SDRD)?

1. Planning of Survey.
2. Formulation of Sample Design.
3. Conducting of Sample Survey.
4. Preparation of Survey Reports.

Select the correct answer using the code below

(i) 1,2 & 4 (ii) 1,2 & 3 (iii) 1,3 & 4 (iv) 2,3 & 4.

Group – B (Answer any six questions) $5 \times 6 = 30$

2. What is simple random sampling? In an SRSWR of size n from a finite population of size N, find an unbiased estimator of the population mean and find the variance of this estimator. Show that $\bar{y}_{wor}$ is more efficient than $\bar{y}_{wr}$. 5

3. What is coefficient of Variation? Determined the sample size such that coefficient of variation of $\bar{y}_{wor}$ should not exceed a given value, say C_0 and find out the smallest sample size needed in this case. Also discuss the situation when N is large. 5

4. Discuss the estimation procedure of population proportion. Show that in case of SRSWOR, $Var(p) = \frac{N-n}{N-1}\frac{PQ}{n}$. 5

5. What do you mean by stratified random sampling? Find out the variance of $\bar{y}_{st}$ in case of proportional allocation. For this sampling, minimize the cost function for the specified variance V_0 and show that

$$n_i = \frac{W_i S_i}{\sqrt{C_i}} \frac{(\sum_{i=1}^{k} W_i S_i \sqrt{C_i})}{(V_0 + \sum_{i=1}^{k} \frac{W_i S_i^2}{N_i})}.$$ 5

6. Show that $Var(\bar{y}_{wor}) : Var(\bar{y}_{sys}) = n : 1$ for the population with linear trend and when $1/k$ is negligible. 5

7. What are the difference between Stratified sampling and cluster sampling? Find out the bias and mean square error of the ratio estimator. When the ratio estimator is more efficient than the sample mean based on SRSWOR ? 5

8. Write short notes on: Statistical System at the Centre, MoSPI Legal Support for Collection of Data. 5

9. Briefly describe how the modern era of Indian official statistics inspired by Rangarajan Commission? 5

10. Discuss about the various divisions of NSSO and their functions. 5

11. Write down any two major publications of each of the following Ministries: 5
a) Ministry of Agriculture and Irrigation b) Ministry of Labour c) Ministry of Finance.

B.Sc. (Honours) Examination, 2018
Semester-VI
Statistics
Course : BSC-61
(Statistical Quality Control)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. (a) Distinguish between process and product control. Give illustrative examples. 3
   (b) Distinguish between allowable and assignable variations. Give illustrative examples. 3
   (c) Let p_n be the probability that the mean of a sample of size n will fall outside the control limits. Obtain the probability that at most x samples are to be taken for r points to go out of control.
2. (a) Explain the basis of Shewart's control chart technique. 4
   (b) Describe, in detail, the construction of control chart for range. 5
3. Describe the situation where modified control chart is used. Derive the limits of modified control charts for mean and fraction defective. 5
4. Give an account of single sampling inspection plan by attributes. With reference to this sampling scheme, derive explicitly the expressions for (1) consumer's risk, (2) producer's risk, (3) average amount of total inspection, and (4) averaging outgoing quality (AOQ). Also derive the OC curve of this plan. 3+7=10
5. Describe a double sampling inspection plan for attributes. Obtain the expression of its OC function. How would you use this OC function to find the plan parameters? 10
6. When a process is said to be capable of producing quality products? Discuss various process capability indices. Give a brief sketch of estimating these indices. 10

B.Sc. (Honours) Examination, 2018
Semester-V
Statistics
Course : BSC-53
(Sample Survey)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. a) Compare the advantages and disadvantages of cluster sampling, two-stage sampling and stratified sampling.
   b) Find the optimal size of first stage and second stage units selected for minimizing the variance of the estimator of population total in two – stage sampling. 4+6=10
2. a) 'Systematic sampling is a mixed sampling scheme' - explain.
   b) Discuss the problem of estimation of the variance of the unbiased estimator of population total in systematic sampling. How can you solve the problem? 3+(2+5)=10
3. a) The units in a population are allocated to two strata with $\frac{N_1}{N_2} = d$ and $\frac{\sigma_1}{\sigma_2} = \lambda$. In WOR sampling from within strata for estimating the population total, let V_e be the variance when the administratively convenient equal allocation $(n_1 = n_2 = \frac{n}{2})$ is used and V_0 be the variance when the optimal allocation is used. Show that
   $$\frac{V_e - V_0}{V_0} = \left(\frac{1-\lambda d}{1+\lambda d}\right)^2$$
   b) Discuss about the use of auxiliary information in estimating the population total and its advantages. 5+5=10
4. a) Define ratio estimator of population total. Find approximate expressions for the bias and MSE of the estimator.
   b) In this context, suggest a ratio-type estimator which is unbiased for the population total. 6+4=10
5. a) What do you mean by double sampling?
   b) Define regression estimator for double sampling and its approximate MSE.
   c) Compare its performance with the regression estimator for single – phase sampling. 2+5+3=10
6. a) A Simple random sample of size $n = n_1 + n_2$ with mean $\bar{y}$ is drawn from a finite population and a simple random subsample of size n_1 is drawn from it with mean $\bar{y}_1$. Show that

P.T.O.

i) $Var(\overline{y}_1 - \overline{y}_2) = S_y^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)$

ii) Var $(\overline{y}_1 - \overline{y}) = S_y^2\left(\frac{1}{n_1} - \frac{1}{n}\right)$

iii) $\text{Cov}(\overline{y}, \overline{y}_1 - \overline{y}) = 0$,

where S_y^2 is the population variance and $\overline{y}_2$ is the mean of the remaining n_2 units in the sample.

b) Mention some advantages of using random number tables. 6+4=10

7. a) Discuss about the basic principles of sample survey.

b) What are the different sources of non-sampling error? 5+5=10